GENERAL PHYSICS AND DIVING PRINCIPLES
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INTRODUCTION

Overview

Technical diving used to be the pervue of just commercial and military divers. Today, highly motivated and well trained recreational divers are pushing diving to new depths, on mixed gases, with sophisticated electronic sensors and dive computers, using modern rebreathers, wearing special exposure suits, in the oceans, lakes, and at high altitude. This new breed of diver receives training from any one of a number of new technical agencies, like Technical Diving International (TDI), International Association Of Nitrox And Technical Divers (IANTD), and Association Of Nitrox Diving Instructors (ANDI), as well as the established recreational agencies, NAUI, PADI, YMCA, SSI, and NASDS. For the technical diver and working commercial diver, this monograph is intentionally both a training tool and extended reference.

Technical diving encompasses a wide spectrum of related disciplines, from geosciences to biosciences, atmospherics sciences to hydrodynamics, medical sciences to engineering sciences, and mathematical physics to statistical analysis. The scope is immense, and so any monograph need be selective, and probably not in depth as possible. And diving physics can be a tedious exercise for readers. Obviously, physiology is an even more complicated mix of physics, chemistry, and biology. Like comments apply to decompression theory, a combination of biophysics, physiology, and biochemistry in a much cloudier picture within perfused and metabolic tissue and blood. Biological systems are so complex, beyond even the fastest and biggest supercomputers for modeling analysis. The marine and geosciences are also beyond comprehensive treatment. Often, tedium relates to a proliferation of equations and deduced results without practical application.

So, selectivity with mathematical application is a direction taken here in narrative. Mathematical equations are kept at definitional level to facilitate description. The hope is to better encapsulate a large body of underlying physical principle in hopefully readable form. Sample problems, with solutions, are included to enhance quantitative description and understanding. Topics are fundamental and chosen in their relevance to technical diving. Bibliographies offer full blown treatments of principles detailed.

Matter

Matter has definite mass and volume, can change form and phase, and consists of tiny atoms and molecules. A gram molecular weight ($gmole$) of substance, that is, an amount of substance in grams equal to its atomic weight, A, possesses Avogadro's number, N_0, of atoms or molecules, some 6.025×10^{23} constituents. Gas molecules are in constant motion. Liquid molecules are free to move and slide over each other, while loosely bound. Molecules in a solid are relatively fixed, but can oscillate about their lattice points.

Matter cannot be created nor destroyed, but it can be transformed by chemical and nuclear reactions. In the most general sense, matter and energy are equivalent. For instance, the nuclear and chemical binding energies of molecules and atoms result from very small mass reductions in constituent particles (mass defect) when in bound states. The postulate of conservation of mass-energy is fundamental, and cannot be derived from any other principle. Stated simply, mass-energy can neither be created nor destroyed. All of observable science is based on this premise.

The concepts of mass and corresponding occupied volume are fundamental perceptions. The mass, m, in unit volume, V, is the mass density,

$$\rho = \frac{m}{V}, \tag{1}$$

and gases are usually the least dense, followed by fluids, and then solids. Weight density is the weight per unit mass. Specific density, η, is the ratio of material density to density of water. States of matter usually have much different densities. Matter interactions are generically termed mechanics.

Motion

Mechanics is concerned with the effects of forces to produce or retard motion (kinetic energy), change position, induce material deformation, or cause chemical and nuclear reactions (potential energy). Forces may be gravitational, nuclear, or electromagnetic in origin. Mechanical properties describe the change in shape of matter when external forces are applied. Examples include the simple bending of a beam, the propagation of sound waves, the permanent deformation of metals into useful shapes, and the flow of liquids and gases around obstacles. For matter in the gaseous state, the usual force is the hydrostatic pressure, and deformation is a change in volume. For matter in the solid state, both tensile and shearing forces come into play to produce deformations.

Time rate of change distance is velocity, v, or, using vector notation,

$$\mathbf{v} = \frac{d\mathbf{s}}{dt} \tag{2}$$

with $d\mathbf{s}$ the inifinitesimal change in position over change in time, dt. Time rate of change of velocity is acceleration, $\mathbf{a}$,

$$d\mathbf{a} = \frac{d\mathbf{v}}{dt} \tag{3}$$

The above set of equations are cast in vector notation, and the addition, subtraction, velocity and particle displacment are are directed processes. To add vectors, place them tip to tail, preserving their magnitudes and directions, and draw the resultant from the tail of the first to the tip of the second vector. Or to subtract vectors, reverse the direction of the subtracted vector, and place it on the tip of the first vector, then proceeding as with vector addition.

Force

Force is a push or a pull. Newton's first law states that a body in motion tends to stay in motion unless acted upon by an unbalanced force. Forces, $\mathbf{F}$, acting upon bodies of mass, m, produce accelerations, $\mathbf{a}$, linked by Newton's second law,

$$\mathbf{F} = m\mathbf{a}. \tag{4}$$

With t the time, the most general form of the force law is,

$$\mathbf{F} = \frac{d\mathbf{p}}{dt}, \tag{5}$$

with $\mathbf{p}$ the momentum, defined in terms of mass, m, and velocity, $\mathbf{v}$,

$$\mathbf{p} = m\mathbf{v}, \tag{6}$$

allowing for changes in mass to generate force. Such situation obviously presents itself in the relativistic case, where mass depends on velocity. Another case where force depends on rate of mass loss occurs with fuel burnup in rocket propulsion systems. Newton's third law states that for every action, there is an equal and opposite reaction. Stated another way, for every applied force, there is an equal and opposite reaction force, a stipulation requiring the conservation of momentum in all reactions.

The rectilinear equations above generalize for the curvilinear case. Angular momentum, $\mathbf{L}$, about some fixed point a distance, $\mathbf{r}$, away, is defined as,

$$\mathbf{L} = \mathbf{r} \times \mathbf{p} \tag{7}$$

and the corresponding torque, $\mathbf{N}$, is then,

$$\mathbf{N} = \frac{d\mathbf{L}}{dt} \tag{8}$$

Obviously, in terms of the force, **F**,

$$\mathbf{N} = \mathbf{r} \times \mathbf{F} \tag{9}$$

A force applied to an element of surface area, at angle, θ, to the surface element normal, generates a pressure, P, given by,

$$P = cos\,\theta \frac{dF}{dA}, \tag{10}$$

with dF and dA scalar elements of force and area. Pressure at a point is equal in all directions, and thus is not a specifically directed (vector) quantity.

Energy

Energy in simplest terms is the ability to do work. Or equivalently, the ability to do work requires an interchange of energy between a system and its surroundings. Energy takes two main forms, kinetic and potential. Kinetic energy is the energy associated with motion. Potential energy is the energy associated with position in a force field. Binding energy is the energy associated with changes in both kinetic and potential energies in bound composite systems, undergoing chemical, nuclear, or molecular interactions. Electromagnetic and acoustical energies are kinetic and potential energies associated with light and pressure waves. Heat energy can be kinetic energy associated with random molecular translations, vibrations, and rotations, or potential energy of frictional surface distortions and stress fatigue, nuclear and chemical reactions, and phase transformations. In all processes known to man, mass-energy is conserved, which is to say that mass can be converted to energy, and vice versa.

In the most general (relativistic) sense, mass and energy are equivalent, as mentioned, which follows as a consequence of the constancy of the speed of light in any inertial frame. An inertial frame is a frame of reference moving with constant velocity (no acceleration). Einstein postulated that the laws of physics are identical for two observers moving with constant velocity with respect to each other (first law of relativity), and that the speed of light, c, is constant independent of relative motion between reference frames (second law of relativity). This requires that the mass, m, of a body moving with speed, v, increases over its resting value, m_0, according to the relativistic equation,

$$m = \frac{m_0}{(1 - v^2/c^2)^{1/2}} \tag{11}$$

for c the speed of light. The corresponding total energy, E, becomes,

$$E = mc^2 \tag{12}$$

and the momentum, **p**, satisfies,

$$\mathbf{p} = m\mathbf{v} \tag{13}$$

as before, but employing the relativistic mass. In the low energy limit, that is, the classical realm,

$$\lim_{v/c \to 0} \frac{m_0 c^2}{(1 - v^2/c^2)^{1/2}} \approx m_0 c^2 + \frac{1}{2} m_0 v^2 \tag{14}$$

so we write the total energy as the sum of rest mass energy, E_0, plus kinetic energy, K,

$$E = m_0 c^2 + \frac{1}{2} m_0 v^2 = E_0 + K \tag{15}$$

with

$$E_0 = m_0 c^2 \tag{16}$$

$$K = \frac{1}{2} m_0 v^2 \tag{17}$$

in the usual (nonrelativistic) sense.

Force, **F**, acting along a pathlength, **ds**, does work, dW,

$$dW = \mathbf{F} \cdot \mathbf{ds}, \tag{18}$$

or, in terms of pressure, P, effecting a volume change, dV,

$$dW = PdV, \tag{19}$$

imparting, or taking, energy to, or from, a system. If there are zero net forces on a system, total energy, $H = K + U$, remains constant, with K kinetic energy and U potential energy. Various forms of the system energy can change, but the total, H, cannot change. If net forces do work on a system and if, when the processes are reversed, the system returns to its initial value of energy, H, the forces are said to be conservative, and the energy of the system is independent of how the work was done. One nonconservative force is friction, since the amount of energy lost to friction by a moving body depends on the distance over which the body slides, and not just on initial and final states. Conservative forces are said to derive from potentials, U, so that we write,

$$\mathbf{F} = -\nabla U, \tag{20}$$

in which case, the total energy, $H = K + U$, is a constant of motion. In a conservative force field, the change in energy associated with initial and final states depends only on initial and final state energies, and is independent of the path chosen between points. Then two (energy) states, i and f, for a conservative transition, are linked according to,

$$H_i = K_i + U_i = H_f = K_f + U_f \tag{21}$$

Potential, U, will depend on position in force fields (gravitation, electromagnetism, strong and weak interactions, and combinations). In the gravitational field of the Earth, we reference the geopotential with respect to position, h,

$$U = mgh \tag{22}$$

with m the mass, g the local acceleration of gravity, and h measured from any convenient Earth reference point in the vertical direction (center, surface, satellite orbit).

Power

Power, J, is the rate of doing work,

$$J = \frac{dW}{dt}, \tag{23}$$

for corresponding small changes in energy and time, dW and dt.

The interactions of matter and energy are sometimes broken down into light, heat, and sound. Macroscopically, this is a classical division, suitably splitting mechanics into major observable categories, but with understanding that each is a detailed science by itself.

ENERGY TRANSFER AND STATE VARIABLES

Light

Light is energy in the form of radiation, equivalently regarded as photons (particles) or electromagnetic packets (waves). Light, regarded as photons in the energy range, 2.5 up to 5.2×10^-19 *joule*, or electromagnetic waves in the wavelength range, 380 up to 800 *nm*, causes sensation of vision. Solar radiation reaching the Earth's surface is peaked in this same spectral range, a range where humans and animals possess sensitive receptors. Light forms a small part of the continuous spectrum of electromagnetic radiation, which encompasses radio waves and infrared radiation at wavelengths longer than light, and ultraviolet, x-ray, gamma ray, and cosmic ray radiation at progressively shorter wavelenghts. As a wave, light is characterized by crossed electric and magnetic field vectors, **E** and **B**.

Electromagnetic waves are transverse, that is, **E** and **B** oscillate in a plane perpendicular to the direction of travel, unlike acoustical waves which are longitudinal and oscillate in the direction of travel. In terms of frequency, f, and wavelength, λ, electromagnetic waves propagating in a vacuum satisfy, $\lambda f = c$, and, treating as photons, light possess particle energy, $\varepsilon = hf$, for h Planck's constant (6.625×10^{-34} *joule sec*). In a vacuum, light (waves or photons) travels at constant speed, c, but in a material medium, however, photons are absorbed and emitted by molecules, slowing down the speed of propagation in the medium.

Refractive index, n, is really a function of wavelength, so that two light beams of different color (wavelength) propagate through materials at different speeds. Across the visible spectrum, differences in refractive indices are small. In glass, 0.009 is the difference between blue and red light indices of refraction. Table 1 lists refractive indices of a few materials.

Table 1. Refractive Indices.

media	refractive index n	media speed of light c/n (m/sec)
vacuum	1.0000	2.99×10^8
air	1.0003	2.98×10^8
glass	1.4832	2.02×10^8
quartz	1.4564	2.05×10^8
steam	1.3178	2.27×10^8
salt water	1.3340	2.24×10^8
pure water	1.3321	2.24×10^8

When light passes from one dielectric medium to another, it is refracted and reflected according to the refractive indices of the media. The relationships between angles of incidence, ϕ, refraction, ϕ', reflection, ϕ'', and the indices of refraction, n and n', assuming $n' > n$, are,

$$\phi = \phi'' \tag{24}$$

and, according to Snell's law,

$$n' \sin \phi' = n \sin \phi \tag{25}$$

At the interface of denser media, there exists a critical angle, ϕ_c, such that for all larger angles of incidence in that media, all light is reflected (grazing incidence),

$$\sin \phi_c = \frac{n}{n'} \tag{26}$$

Water is transparent to light. Although a glass of water seems to allow all the light to pass through, it is obvious that as one goes deeper underwater, it gets darker. In the ocean, with so much water available, the amount of light energy absorbed becomes important. Water clarity and lack of turbidity are also primary factors in determining light penetration in different regions, or layers. Thus, it is difficult to determine at what depth water becomes dark. Some indication of light penetration is the depth at which microscopic plants exist underwater, because marine plants, like land plants, require light for photosynthesis. The vertical region in the ocean where light exists is called the euphotic zone, existing from the surface down to where only 1% of the light remains. The lower limit varies from 45 fsw along the coasts, down to as much as 500 fsw in the clear tropical zones. As one descends, white sunlight is selectively absorbed, starting with the red part of the spectrum, and then continuing to the green and blue parts. Colors, such as red, perceived in fish and other creatures underwater, do not come from surface light. Pigments in these creatures absorb the remaining blue-green light, and then remit the light as red. At a depth of 33 fsw, little or no color distinction is possible. There are no shadows, and light seems to be coming from all around. At 330 fsw, visibility is limited to a few feet. At 950 fsw, all is quite dark.

Unlike sound waves encountering density interfaces, light transmission through opaque dielectric interfaces is slightly attenuated, with energy passing easily from one media to the other. Because of refraction, however, perceived images of source objects differ across the media interface. Such refractive phenomena change image size and relative position, the study of which is called optics.

Optics

Optics deals with ray phenomena that are not dependent in any way on the wave or quantum behavior of light. In geometrical optics, light travels along straight lines, or rays, in homogeneous media, which are bent at the interfaces separating media, or curved in media with variable refractive indices. At any point along a fan of rays emitted by an object point source of light, there is a surface everywhere perpendicular to the rays, called the wave front. The wave front is the locus of points reached by light after a given time along all possible ray paths. If the wave front emerging from a lens or other optical interface is a true sphere, a perfect image will be formed. Any departure from a true sphere represents the presence of optical aberrations, or, more simply, image distortions. An optical system consists of an assembly of mirrors, lenses, prisms, and apertures, usually with spherical surfaces to facilitate precise image formation. The human eye is an optical system consisting of lenses, apertures, and image forming planes.

Each ray from an object point, after passing through an optical system (such as the eye), strikes a specified image plane at a single point, with all such points for all possible rays passing through the system constituting the geometrical image of the source as formed by the optical system. While the number of rays are infinite, only a few rays, strategically chosen with regard to the optical system, are actually traced in an image assembly called a spot diagram. The spot diagram represents an outline picture of the image produced by the optical system, but lacking fine structure caused by light wave interference and diffraction. In spite of microstructural limitations, the simple ray tracing technique can quantify gross relationships between souce and image sizes, distances, focal lengths, and refractive indices of optical media.

Refraction of paraxial rays (very nearly normally incident) is a good example of simple ray tracing techniques in optical applications. The ratio of image to object distance, σ, is termed shortening, while the ratio of image to object height, μ, is the lateral magnification. For paraxial bundles of rays, the dispersion is small and the bundle is clustered at near normal incidence. Always, $\sigma\mu = 1$. Objects underwater, viewed at the surface, appear larger and closer than their actual size and position. The shortening is 3/4, while the lateral magnification is 4/3, taking $n = 4/3$ for water, and $n' = 1$ for air. The opposite occurs underwater, when viewing an object above the surface. Underwater viewing of surface objects is also limited by the critical angle, ϕ_c. Outside the viewing cone, limited by ϕ_c underwater, no surface images can be transmitted through the water to the eyes.

The eyes focus using paraxial rays. The ability to accommodate angular spread in the paraxial bundle is called peripheral vision. The greater the ability to accommodate angular dispersion in rays striking the eye, the greater is the corresponding peripheral vision. Cutting off the most widely dispersed rays in the bundle reaching the eyes, for instance, with a mask underwater, causes tunnel vision, or the perception of a brightly illuminated foreground, and dark peripheral background

The refraction and focusing of overhead sunlight by wave motion produces the pattern of light ripples often seen on sandy bottoms below shallow, clear water. Wave crests act like converging lenses, focusing light rays into spatial regions of higher intensity, while the troughs act like diverging lenses, defocusing light rays into spatial regions of lesser intensity.

Sound

Any change in stress or pressure leading to a local change in density, or displacement from equilibrium, in an elastic medium can generate an acoustical wave. Acoustics is concerned with fluctuations in mechanical properties characterizing the state of matter, such as pressure, temperature, density, stress, and displacement. Primary acoustical measurements determine the magnitude and wave structure of one of these mechanical properties, whereas secondary measurements characterize the propagation speed and the rate of dissipation of acoustical energy.

The time averaged energy density, I, of an acoustical wave is a sum of kinetic and potential (strain) contributions, and can be written, $I = 2\pi^2 f^2 U^2 \rho u$, for u sound speed, ρ material density, f frequency, and U wave amplitude. At an interface, energy is both transmitted and reflected. The transmitted wave amplitude, TU, and reflected wave amplitude, RU, depend on the density and acoustical speed in both media. Across dissimilar interfaces, very little energy, emanating as an acoustical signal in either water or air, is transmitted. For an air-water interface, we have

$$T = .0081 \ , \tag{27}$$

$$R = .9919 \ , \tag{28}$$

in approximately both cases (air-water, or water-air propagation), using nominal values of water and air densities, and acoustical speeds. Corresponding ratios of transmitted to incident intensity, and reflected to incident intensity, measures of the acoustical energy transmitted and reflected across the boundary, are given by

$$\frac{I_T}{I} = T^2 = .0092 \ , \tag{29}$$

$$\frac{I_R}{I} = R^2 = .9839 \ , \tag{30}$$

These results parallel electromagnetic wave propagation across a metallic-dielectric (conducting-nonconducting) interface.

Sound propagation is but one aspect of acoustics. When we speak, we utter sound. Someone nearby hears the sound. In studying the production and reception of sound, and transmission through media, acoustics is a discipline of physics, but speech and hearing obviously invoke biological elements and processes. When speaking, a slight disturbance is produced in the air in front of the mouth, a compression resulting in a change in air pressure near 1 $dyne/cm^2$. Since air is an elastic medium, it does not stay compressed and expands again passing on the disturbance to its neighbor. That neighbor in turn passes the disturbance on to its next neighbor, and so on, resulting in a pressure fluctuation that moves through the air column in the form of a sound (acoustical) wave. Reaching the ear of an observer, the disturbance moves the eardrum, which in turn displaces the little bones in the middle ear, communicating motion to the hair cells in the cochlea, with the ultimate biophysical interpretation of the hearing of the sound by the brain.

A great bulk of data gathered about the ocean bottom, and other underwater objects, uses sound navigation and ranging (sonar). Sonar may be active or passive. Passive sonar equipment listens to noises underwater, and can determine presence and relative direction of sound sources. Active sonar, or echo sounding, acts like radar, sending out an acoustical signal which is reflected back to a receiver. If sounding from the bottom, the depth is equal to 1/2 the time for the signal to leave and return, multiplied by the speed of sound in water, about 4,950 ft/sec. Ships tracing out prescribed paths can continuously map the bottom with sonar. is sinusoidal, as with light waves. Sound speeds in various media are tabulated in Table 2, at 0 C^o and 1 atm.

Table 2. Sound Speeds At Standard Temperature And Pressure.

media	sound speed u (m/sec)
vacuum	0
air	333
steel	5302
copper	3292
parafin	1395
wood	2984
salt water	1452
pure water	1461

Heat

In thermodynamics, heat denotes the quantity of energy exchanged by thermal interaction of any system with its environment. For example, if a flame is applied to a cool metal plate, the energy content of the plate increases, as evidenced by its temperature increase, and we say that heat has passed from the flame to the plate. If energy losses to the surrounding air can be ignored, the heat transferred from the flame is equal to the energy gain of the plate. In more complex processes, involving mechanical as well as thermal interactions, the heat transferred is more difficult to identify. Thermodynamics focuses on the controlled and slow evolution of heat, energy, and entropy, and the distinctions between them in mechanical systems. While heat is a tenuous concept, linked to observables such as internal energy change and external work, we often deal with systems at different temperatures, exchanging heat in the absence of mechanical interactions, or external forces. Specific heat, c, measures change in heat capacity, dQ, for corresponding change in temperature, dT, per unit mass, m, of substance. At constant pressure, the specific heat is denoted, c_P,

$$c_P = \frac{1}{m}\left[\frac{dQ}{dT}\right]_P \tag{31}$$

while at constant volume, the specific heat, c_V, is similarly written,

$$c_V = \frac{1}{m}\left[\frac{dQ}{dT}\right]_V \tag{32}$$

Generally, it is c_P that concerns us as divers and underwater. The molal specific heat is the heat capacity per unit mole (n replaces m). Heat, then, is the energy exchanged between parts of mechanical systems at different temperatures. Three fundamental and well known mechanisms include convection, conduction, and radiation. In practical situations, near standard temperatures and pressures, heat exchange usually involves the first two, conduction and convection. Radiative transfer underscores fairly high temperatures.

Heat conduction is the exchange of heat from one body at a given temperature to another body at a lower temperature with which it is in contact. Transfer of molecular kinetic energy occurs directly by molecular impacts or collisions. Heat conduction is governed by Fourier's law,

$$\phi = -K\nabla T, \tag{33}$$

with, ϕ, heat flux, K, conductivity, and, T, temperature.

Heat convection is a special case of conduction that occurs when a fluid or gas flows past the outer boundary of a system. Then the determination of K involves solving the fluid equations of a viscous, heat conducting fluid or gas, coupled to the heat flow equations in the system. Table 3 summarizes specific heats, conductivities, and corresponding densities for a cross section of materials.

Radiative transfer is a different mechanism completely from conduction and convection. The mechanism is electromagnetic wave emission from a heated surface, with the spectrum of wavelengths a complex function of surface temperature. For a point (idealized) source at temperature, T, the radial (isotropic) heat flux, ϕ, is given by the Stefan-Boltzmann relationship,

$$\phi = \sigma T^4, \tag{34}$$

for T the temperature, and σ the radiation constant $(5.67 \times 10^{-8} watt/m^2\,K^{o4}$. The most complex heat transfer phenomena are those in which extended physical systems interact by combinations of the above, in addition to phase transformations such as boiling, condensation, or solidification.

Table 3. Specific Heats, Conductivities, And Densities.

material	specific heat c_P $(cal/g\,C^o)$	conductivity K $(cal/sec\;cm\;C^o)$	density ρ (g/cm^3)
air	.242	.0001	.00024
iron	.121	.0858	16.623
aluminum	.207	.5375	2.721
polyethylene	.912	.6939	.925
neoprene	.381	.0004	.189
glass	.135	.0025	2.312
salt water	.949	.0013	1.025
pure water	1.000	.0014	1.000
alcohol	.653	.0010	.791

Radiation is absorbed in passing through matter, and the fraction absorbed is characteristic of the material. The ratio of absorbed to incident radiation at a certain wavelength is called the absorptivity, α, and depends on the wavelength. A body with absorptivity equal to one is called a *black* body. Perfect black bodies do not exist in nature, but there are many approximate black bodies, especially in the infrared, or long wavelength, region. Of the incident radiation that is not absorbed, part is reflected and part is transmitted. The ratio of reflected to incident radiation is called the reflectivity, ρ, and the ratio of transmitted to incident radiation is called the transmissivity, τ. Obviously, the three quantities are related by,

$$\alpha + \rho + \tau = 1. \tag{35}$$

For a black body, $\rho = \tau = 0$, and $\alpha = 1$. A molecule which absorbs radiation at a particular wavelength is also able to emit radiation at the same wavelength. The emissivity, ε, is defined to be the ratio of emitted radiation to the maximum possible at a given temperature, and by Kirchhoff's law,

$$\varepsilon = \alpha. \tag{36}$$

Equation Of State

The relationship between pressure, volume, and temperature for any substance is called the equation of state (EOS). In the case of solids and liquids, equations of state are typically quite complicated, mainly because molecular interactions in solids and liquids are extended (long range). Gases, however, present a simpler situation. Interactions of gas molecules are localized (short range), compared to solids and liquids, and the corresponding equation of state reflects the point nature of interactions.

Long before kinetic theory and statistical mechanics provided the molecular basis for gas laws, chemists (and probably alchemists) deduced that, under pressure, P, volume, V, and temperature, T, changes, to good order of the day,

$$\lim_{P \to 0} \frac{PV}{nT} = R, \tag{37}$$

for n the number of moles of the gas, with R a constant, and temperature, T, measured on an absolute (K^o) scale. Obviously, their range of investigation was limited in phase space, but it is still interesting to note that all gas laws reduce to the simple form in the limit of low pressure, for any temperature.

With this subset of definitions, the relationship between pressure, volume, temperature, and number of moles of gas is ideal gas law, and R the universal gas constant $(8.317\ joule/gmole\ K^o)$. If temperature is kept constant, pressure increases linearly with inverse volume. If pressure is kept constant, the volume varies linearly with temperature, or, if the volume is kept constant, the pressure increases linearly with temperature. The fact that real gases approximate this behavior has been known for centuries, in fact, the quantifications are often called the laws of Boyle, Charles and Gay-Lussac, after the Eighteenth century investigating chemists.

Rheology

Rheology is the interdisciplinary study of the deformation and flow of material under internal and external forces. Rheology tries to correlate macroscopic response and flow of solids, liquids, and gases with constitutive equations spanning atomic, molecular, intermolecular, and broader domain scales. Irreversible processes, such as macroscopic flow, heat generated by internal friction, mechanical aging, fatigue, solid deformation, shearing, and stressing can be collectively quantized through constitutive equations. The mechanical relationships, describing the change in shape or flow of matter under applied forces, are also called *material* properties.

Deformation

A solid is elastic if the amount of deformation is directly proportional to the applied force, implying that the deformation process is reversible and independent of the way the force is applied. A solid is inelastic if the displacement depends on the rate, or direction, of the applied force. Interest in the elastic and inelastic properties of matter date back to Galileo.

For most metals and ceramics, the rate dependent effects are small, but play an important role in the dissipation of oscillational energy, causing damping of vibrations in machines and oscillating mechanical systems. In plastics and rubbers, inelastic contributions to deformations are large, and these types of materials are termed viscoelastic. The categorization, plastic solid, or plastic deformation, is appropriate for materials in which deformation is a nonlinear, irreversible function of the applied force. Examples include the permanent deformation of metals by large forces, the response of organic polymers, and glass at high temperature. For perfectly elastic solids, stress and strain are completely reversible, so that the energy stored in the solid under stress is returned when the stress is removed. In such case, a stressed solid will vibrate and oscillate between deformed and relaxed state indefinitely.

Friction and viscosity are certainly dissipative forces, tending to convert kinetic energy into mostly heat and potential energy that is not recoverable. Frictional and viscous forces impart irreversibility to physical processes, contributing to overall increase in entropy. Perpetual motion machines, indefinite oscillations, and perfect multidirectionality of physical phenomena are precluded because of dissipative mechanisms. Yet, without frictional and viscous forces, we could not walk, drive cars, nor swim underwater with fins.

Friction And Tribology

Friction is the tangential force necessary to overcome resistance in sliding contacting surfaces against each other, under a normal force pressing the surfaces together. Friction is mainly a surface phenomenon, depending primarily on conditions at the interfacial surfaces. By definition, friction is the ratio of the magnitudes of the required moving force, F, to the normal (load) force, N, and takes the form,

$$\mu = \frac{F}{N}, \tag{38}$$

with the coefficient of friction, μ, ranging from small to large values. For lubricated surfaces, μ ranges from 0.001 to 0.2, for dry surfaces, μ, varies between 0.1 and 2.0, while for ultraclean surfaces, μ becomes very large. For ultrasmooth surfaces, μ is large because of large cohesive forces, while for very rough surfaces, μ is large because of high asperity interlocking.

The maximum value of frictional force required to start sliding is known as *static* friction, while the amount of frictional force necessary to maintain sliding is known as *kinetic* friction. Static friction is always slightly greater than kinetic friction. Some static coefficients of friction for metals are listed in Table 4 below. Kinetic coefficients are no more than 5% to 10% less.

Table 4. Coefficients Of Static Friction

metal	against itself	against steel
aluminum	1.3	0.6
brass	1.4	0.5
bronze	1.2	0.4
copper	1.3	0.8
iron	0.4	0.4

Contact between surfaces that are dry, and ordinarily rough, usually involves the tips of tall asperities. thus, total contact area is only a small fraction of the entire interfacial area. Tips adhere to opposing surfaces, and must be sheared if motion is to occur. Total force requisite to shear these junctions is roughly the product of the shear strength of the materials times the area of all junctions at the onset of sliding.

Wear (tribology), concerned with the loss or transfer of material in contact, results from many interactive frictional forces, including adhesion, abrasion, corrosion, fatique, and worse, combinations of all. The volume of wear (material lost), w, is proportional to the applied normal load, N, distance moved or slid, x, and inversely proportional to the material hardness, β, so that,

$$w = \frac{kNx}{\beta}, \tag{39}$$

with k the proportionality constant, obviously a function of many variables. Lubrication attempts to mitigate wear by imposing films of foreign substances between contacting bodies, with films solid (graphite), fluid (oil), or chemically active substances. Elastohydrodynamic lubrication occurs in highly loaded assemblies with changes in fluid viscosities under high pressure and temperature, seen, for instance, in multiple viscosity oil for car engines and compressors.

Viscosity

Fluid flow is invariably accompanied by drag, that is, mechanical work is expended to keep the fluid in motion, and is then converted into heat. The effect is linked to viscosity, or internal fluid friction as it is termed in analogy to material properties. Viscosity arises in fluids and gases as a result of momentum transfer between adjacent layers of molecules, simply, shear forces resulting from velocity differences between molecules in interacting layers. Velocity differences can arise through applied forces, temperature differences, boundary effects, or local turbulence and mixing. Like friction, viscosity is dissipative, tending to resist motion, or changes in motion.

In gases, viscosity is proportional to the square root of absolute temperature, and is essentially independent of pressure. For actual gases, viscosity is indeed constant over a wide range of pressures, somewhere in the range of 0.01 atm up to 10 atm. In liquids, on the other hand, viscosity falls off rapidly with increasing temperature. Additionally, viscosity in liquids has a short range, intermolecular force component.

Measurement of viscosity is simple, conceptually. Two plates of cross sectional area, A, separated by a distance, Δx, are placed in a fluid. The force, F, required to drag one plate with velocity, Δv, with respect to the other, in parallel direction, defines the viscosity, X, through the relationship,

$$F = \frac{XA\Delta v}{\Delta x}. \tag{40}$$

The statement above assumes that the shear process does not alter the gas or fluid structure. Certainly for gases this poses no problems, but for fluids, this may not always be true. For instance, within polymers, the shearing and flow result in partial alignment of elongated molecules.

Shocks

Shocks are wave disturbances propagating at supersonic speeds in materials, characterized by rapid rise in local pressure, density, and temperature in frontal regions. Shock waves are generated by the sudden release of large amounts of energy in a small region, for instance, detonations in high explosives, passage of supersonic aircraft in the atmosphere, or discharges of lightning bolts in a narrow air channel. Shock waves, not sustained in propagation, lose energy through viscous dissipation, reducing to ordinary sound (acoustical) waves.

Detonation waves are special types of self-sustaining shock waves, in which exothermal reactions move with supersonic speed into the undetonated material, compressing, heating, and igniting chemical reactions that sustain shock propagation. The detonation process usually requires a shock wave to initiate reactions. Deflagrations, or flames, differ from shocks and detonation waves because deflagrations propagate at subsonic speeds.

A unique feature of shock wave propagation in gases is the high shock temperatures attainable, near 15,000 K^o. Such high temperatures are very useful for the study and application of shock tubes to measurements of reaction rate processes in science and aeronautics. Measurements of chemical reaction, vibrational relaxation, dissociation, and ionization rates have been effected with shock tubes, over large temperature ranges.

Modified shock tubes can be used as short duration wind tunnels, so to speak, producing high Mach number ($\mu = 16$), high temperature ($T = 6,000\ K^o$) environments replicating the gas dynamics encountered by missiles and reentry vehicles (RVs). Conventional wind tunnels are constrained by Mach numbers approximately half shock tube Mach numbers.

The shock equation of state, simply the relationship between pressure and volume for given shock speed, has been established for many materials, and up to pressures of 10 $Mbar$. Pressures and densities attained in shock compressed geologic material are comparable to those found in the Earth, and have provided valuable data for geophysical analysis. Volcanism, plate faulting, and marine disturbances generate geological shocks of enormous potential destructive force, and an accurate assessment of their propagation characteristics in the Earth is an important component of seismology and geophysics. Thermodynamics, like rheology, deals with macroscopic properties of extended matter, such as density and pressure, where temperature is a significant variable. Thermodynamics provides a complete description of these properties under conditions of equilibrium, and offers a starting point for investigation of nonequilibrium phenomena such as hydrodynamics, transport, and chemical reactions. Collectively, thermodynamics relates mechanics to heat and temperature changes, assigns directionality to physical processes, and serves as the basis for descriptions of macroscopic interactions. Thermodynamics grew naturally out of early studies of temperature.

THERMODYNAMICS

Thermometry

While thermometry is concerned with heat measurements and fixed calibration points for instruments, some indicated in Table 5 below, what thermometers measure is the average kinetic energy, $\bar{\varepsilon}$, of the molecular ensemble, the essence of temperature. Typically, thermometers employ linear or logarithmic scales, most often using two (sometimes three) calibration points. In the 1,000 to 5,000 K^o temperature range, at 1 atm pressure, all condensed phases (solid and liquid) are unstable against the gaseous phase. There are no known stable solids above 4,200 K^o, the approximate melting point of a mixture of tantalum carbide and hafnium carbide. Stable liquids do exist over the entire range, although not extensively studied. The normal boiling point of tungsten, for example, is about 6,200 K^o. Although intermolecular forces responsible for the stability of solids and liquids begin to weaken as temperature is increased from 1,000 to 5,000 K^o, chemical valence binding is still of considerable importance in the gas phase. Molecular species that are unstable at room temperatures are sometimes found in conditions of equilibrium in high temperature vapors. In the 5,000 to 10,000 K^o range, no stable molecules can exist in the gas phase. At temperatures near 10,000 K^o, atoms and ions can exist together, while above 10,000 K^o appreciable numbers of free electrons are present. At 50,000 K^o, mostly electrons and bare nuclei persist. In the 100,000,000 K^o region, like charged ions in a plasma possess sufficiently high collisional energy to overcome mutual Coulomb repulsion, supporting, in the case of deuterium and tritium, fusion.

Table 5. Temperature Calibration Points.

calibration point	Kelvin (K^o)	Fahrenheit (F^o)	Centigrade (C^o)
absolute zero	0	-460	-273
hydrogen triple	14	-434	-259
neon boiling	27	-410	-246
oxygen boiling	90	-297	-183
water triple	273	32	0
water boiling	373	212	100
sulfur boiling	717	831	444
gold freezing	1336	1945	1063

When reference points, such as freezing and boiling, are known, it is simple to construct thermometers, devices which interpolate and extrapolate over ranges near the reference points. Both linear and logarithmic forms are employed. Denoting the freezing point calibration, X_i, at temperature, T_i, and then boiling point calibration, X_s, at temperature, T_s, linear and logarithmic temperature scales are given by,

$$\frac{T-T_i}{T_s-T_i}=\frac{X-X_i}{X_s-X_i} \tag{41}$$

and,

$$\frac{T-T_i}{T_s-T_i}=\frac{\ln\ (X-X_i)}{\ln\ (X_s-X_i)} \tag{42}$$

with T the temperature for reading X. Not unexpectedly, standard Kelvin, Centigrade (Celsius), Rankine, and Fahrenheit temperatures are defined on linear scales,

$$F^o=\frac{9}{5}C^o+32\ \ , \tag{43}$$

$$K^o=C^o+273\ \ , \tag{44}$$

$$R^o=F^o+460\ \ . \tag{45}$$

First And Second Laws

The first law of thermodynamics is really a statement of conservation of energy in any system. Denoting the internal energy of the system, U, the net heat flow into the system, Q, and the work, W, done on the system, the first law requires that infinitesimal changes dQ, dU, and dW satisfy the balance,

$$dU=dQ-dW, \tag{46}$$

The second law requires an ordering variable, S, called entropy, so that for any process, the heat transferred, dQ, takes the form,

$$dQ=TdS, \tag{47}$$

with $dS\geq 0$. The requirement that the entropy change, dS, associated with the process must be greater than or equal to zero imparts directionality to the process. Combining first and second laws, considering only mechanical work, $dW=PdV$, we see that,

$$dU=TdS-PdV. \tag{48}$$

In mechanics, energy and momentum are usually introduced as derived concepts. Advanced treatments introduce energy and momentum as fundamental quantities. Similarly, in thermodynamics, internal energy and entropy may be introduced as fundamental quantities, instead of pressure, volume, and temperature.

Phase Transformations

Every substance obeys an equation of state, some fundamental relationship between pressure, temperature, and volume. That of ideal gases is a simple example. Real substances can exist in the gas phase only at sufficiently high temperatures. At low temperature and high pressures, transitions occur to the liquid and solid phases. Behavior is interesting, especially for a substance like carbon dioxide that contracts on freezing. Inspection of a phase diagram shows that there exist regions in which the substance can exist only in a single phase, regions termed solid, liquid, vapor, and gas. A vapor is just the gas phase in equilibrium with its liquid phase. In other regions, designated solid-vapor, solid-liquid, and liquid-vapor, both phases exist simultaneously. Along a line called the triple line, all three phases coexist. In the winter, water, ice, and water vapor coexist at $0.0098\,C^o$.

The Clausius-Clapeyron equation relates pressure, temperature, volume, and heat of transformation along the solid-liquid, solid-vapor, and liquid-vapor equilibration lines, according to,

$$\frac{dP}{dT}=\frac{l}{T\Delta v}, \tag{49}$$

with l the appropriate heat of transformation, and Δv the difference in the specific phase volumes at temperature, T. The equation describes the reversible proceses of condensation-vaporization, freezing-melting, and accretion-sublimation, that is, processes proceeding in either direction with the same latent heats of transformation. At the triple point, the latent heats of transformation are additive, specifically, the heat of sublimation equals the sum of the heats of vaporization and melting. For water, the heat of melting is 80 cal/g at 0 C^o, while the heat of vaporization is 540 cal/g at 100 C^o and standard pressure $(1\ atm)$.

Vapor Pressure

Liquids tend to evaporate, or vaporize, by releasing molecules into the space above their free surfaces. If this is a confined space, the partial pressure exerted by released molecules increases until the rate at which molecules return to the liquid equals the rate at which they leave the liquid surface. At this equilibrium point, the vapor pressure is known as the saturation pressure.

Molecular evaporation increases with increasing temperature, hence the saturation pressure increases with temperature. At any one temperature, the pressure on the liquid surface may be higher than this value, but it cannot be lower. Any slight reduction below saturation pressure induces the very rapid rate of evaporation called boiling.

Saturation vapor pressures of known liquids vary widely. Table 6 lists saturation vapor pressures for a number of liquids. At 70 F^o, vapor pressures of mercury and gasoline are seen to differ by a factor of 10^5 approximately.

Table 6. Saturated Vapor Pressures.

liquid	temperature (F^o)	vapor pressure (lbs/in^2)
mercury	70	0.000025
mercury	320	0.081
water	200	7.510
water	70	0.363
water	32	0.089
kerosene	70	0.492
alcohol	70	1.965
gasoline	70	4.439
ammonia	200	794.778
ammonia	50	89.190
ammonia	-100	1.238

ELECTRODYNAMICS

Coulomb's Law

Electrodynamics is the study of charged particles and their associated electrical and magnetic field interactions. The word was coined by Ampere in 1850 to describe all electromagnetic phenomena. The comprehensive description of all electromagnetic phenomena, embodied in Maxwell's equations, is another crowning achievement in science.

The coupling of thermodynamics and electrodynamics is the basis of plasma physics, the study of high temperature matter composed of charged particles. Stellar and interstellar matter is mostly in the plasma state, as is matter in the upper atmosphere (magnetosphere, ionosphere), flames, chemical and nuclear explosions, and electrical discharges. Matter in a controlled thermonuclear reactor would also be in a plasma state, and the study of fusion as a source of energy and power has led to extensive knowledge and advances in plasma physics. In terms of gross properties, plasmas differ from nonionized gases because of their high electrical and thermal conductivity, unusual dielectric and refractive properties, their emission of electromagnetic radiation, collective, long range particle interactions due to the Coulomb force, and very high temperatures.

Electrodynamics is specifically a study of charges in motion, the associated electric and magnetic fields fields produced, and their interaction with, and in, matter. The fundamental entity is electrical charge, and only electrical charge, since corresponding magnetic poles have not been found to date. Electrodynamics describes moving charges and time varying fields, while electrostatics and magnetostatics are concerned with stationary charges and constant fields in time, obviously a subcase. Electrical charge is a property of matter, first observed in ancient Greece in materials we now call dielectrics. Centuries ago, it was noted that amber, upon being rubbed, attracts bits of straw and lighter objects. The Greek word for amber is electron. That electrified bodies attract and repel was noted by Cabeo in the early 1700s, while du Fay and Franklin denoted these two types of electricities, positive and negative, a convention still holding today, and established the notion that charge can be neither created nor destroyed (conservation of charge in physical processes).

Two charges, q and Q, attract (or repel) each other with force, $\mathbf{F}$, given by the Coulomb law,

$$\mathbf{F} = -\kappa_0 \frac{qQ}{r^3}\mathbf{r} \tag{50}$$

for r the distance, κ_0 the Coulomb constant $(8.91 \times 10^9 \ m/f)$, and $\mathbf{r}$ the separation vector. A charge, q, moving with velocity, $\mathbf{v}$, through electric and magnetic fields, $\mathbf{E}$ and $\mathbf{B}$, experiences a Lorentz force, $\mathbf{F}$, from both fields,

$$\mathbf{F} = q(\mathbf{E} + \mathbf{v} \times \mathbf{B}). \tag{51}$$

Maxwell's Equations

Maxwell's equations are four partial differential equations relating electric field, $\mathbf{E}$, magnetic field, $\mathbf{B}$, current density, $\mathbf{J}$, and charge density, ρ. Defining the displacement, $\mathbf{D}$, and magnetic intensity, $\mathbf{H}$,

$$\mathbf{H} = \frac{\mathbf{B}}{\mu} \ , \tag{52}$$

$$\mathbf{D} = \varepsilon \mathbf{E} \ , \tag{53}$$

with ε and μ the material permittivity and permeability, we can write Maxwell's equations,

$$\nabla \cdot \mathbf{D} = \rho \ , \tag{54}$$

$$\nabla \times \mathbf{E} = -\frac{\partial \mathbf{B}}{\partial t} \ , \tag{55}$$

$$\nabla \cdot \mathbf{B} = 0 \ , \tag{56}$$

$$\nabla \times \mathbf{H} = \mathbf{J} + \frac{\partial \mathbf{D}}{\partial t} \ . \tag{57}$$

The relationship between **H** and **B** is analogous to the relationship between **D** and **H**, that is, **H** and **D** depend only on the source of the fields, while **B** and **E** also depend on the local material properties. Thus, **B** and **E** are fundamental, but **H** and **D** can be easier to employ in applications.

In a conductor, with conductivity, σ, permeability, μ, and permittivity, ε, current density, $\mathbf{J}$, is linearily coupled to the electric field, $\mathbf{E}$, by Ohm's law,

$$\mathbf{J} = \sigma \mathbf{E} \ , \tag{58}$$

serving as a corollary to Maxwell's equations. The current driving potential, V, in the conductor also satisfies the electromotive generalization of Ohm's law,

$$V = iR \ , \tag{59}$$

with R the electrical resistance, and i the current. Similarly, an electrostatic potential, V, is generated when a conductor cuts through magnetic field lines, that is, the magnitude of the electromotive force is given by,

$$V = \frac{\partial \phi}{\partial t} \ , \tag{60}$$

with,

$$\phi = \int \mathbf{B} \cdot \mathbf{dA} \ , \tag{61}$$

and **dA** the area swept out by the conductor in cutting magnetic field lines.

Conservation of charge demands that the charge density, ρ, and current density, $\mathbf{J}$, are related by a continuity equation,

$$\frac{\partial \rho}{\partial t} + \nabla \cdot \mathbf{J} = 0 \ , \tag{62}$$

which is just a simple statement that any increase, or decrease, in charge in a small volume must correspond to a flow of charge into, or out of, the same volume element. Electrostatics is defined by the condition,

$$\frac{\partial \rho}{\partial t} = 0 \ , \tag{63}$$

while magnetostatics similarly requires,

$$\nabla \cdot \mathbf{J} = 0 \ , \tag{64}$$

Magnetic materials have traditionally been considered as elements, alloys, or compounds permitting ordered arrangements, or correlations, among electron magnetic moments or spins. Net magnetic polarization can be ferromagnetic, in which all spins are aligned parallel, antiferromagnetic, in which neighboring spins are aligned antiparallel, or ferrimagnetic, in which spins of two dissimilar atoms are aligned antiparallel. Metals such as iron, cobalt, and nickel are ferromagnetic, while manganese and chromium are antiferromagnetic. The temperature necessary to induce a phase transition from an unordered magnetic state to a magnetically ordered state is the Curie temperature, whether ferrromagnetic or antiferromagnetic in the final state. The permanent properties of such materials are useful in magnetic devices, such as computers and transformers.

An essential difference between electric and magnetic interactions appears in the direction of the force. The electrical force acts in the direction of motion, while the magnetic force acts normal to the direction of motion. Hence the magnetic force can only change direction of the moving charge, but cannot do work on it. Interestingly, both the Coulomb and Ampere laws exhibit an inverse square dependence on the separation of source and field point.

Plasmas

Plasma physics is the physics of ionized gases, and is a relatively new science. Not until development of the electrical power industry were controlled experiments on ionized gases possible, so plasma physics is some 100 *yr* old. Studies at the turn of the century of gas discharges and radio propagation off the ionosphere, along with impetus for controlled thermonuclear reaction programs in the 1950s, fueled study of the complex mechanisms attending plasma interactions. The discovery of the solar wind and Van Allen radiation belts in the 1960s provided much data for integration of plasma theory and experiment. Plasmas are complex, exhibiting fluid turbulence and collective motion, linear and nonlinear behavior, and wave and particle motions.

Plasmas exhibit a state of matter in which a significant number, if not all, of the electrons are free, not bound to an atom or molecule. Practically speaking, matter is in the plasma state if there are enough free electrons to provide a significant electrical conductivity, σ. Usually, only a small fraction of electrons need be free to meet this criterion. Collisional ionization, caused by energetic thermal motions of atoms at high temperature, is the source of large numbers of free electrons in matter. Large densities of free electrons are also found in metals at solid densities, independent of the temperature, accounting for the electrical conductivity of metals ar room temperatures and lower. Most terrestrial plasmas, excepting metals which are not plasmas, are very hot and not very dense. The plasma state is the highest temperature state of matter, occuring certainly at much higher temperatures than the gaseous state. Plasmas are hot, ionized gases. Plasmas in discharge tubes, for instance, have subatmospheric densities on the whole. On the cosmological scale, most of the matter in the Universe is thought to reside in stellar interiors, where the density is so high that it is called a stellar plasma independent of its temperature. Of course, the temperature is also so high that only the plasma phase could exist.

In the laboratory, much research interest centers on economical means to exploit plasmas for energy production. Borrowing from what we already know of energy production in solar and stellar plasmas, one focus is fusion energy production in thermonuclear fuels, such as deuterium and tritium.

Fusion Energy

Fusion processes in the solar plasma are responsible for energy radiated to the Earth. For the past four decades, scientists have pursued the dream of controlled thermonuclear fusion. The attraction of this pursuit is the enormous energy potentially available in fusion fuels, and the widely held view of fusion as a safe and clean energy source. The fusion reaction with the highest cross section as a reaction process,

$$D + T \rightarrow n + He \ , \tag{65}$$

releases some 17.6 MeV of energy, denoting deuterium, D, tritium, T, neutron, n, and helium, He. To produce fusion reactions in a deuterium-tritium plasma, very high collisional temperatures are necessary to overcome the Coulomb repulsion between interacting nuclei, and the plasma must be confined for long time scales so that many fusion collision reactions can take place to make the process economically feasible. Temperatures near 3 keV ($3.47 \times 10^7\ K^o$) are necessary for plasma ignition and sustained thermonuclear burn.

Development of an economically viable fusion reactor would literally give us the energy equivalent of oceans of oil. Because seawater contains about 40 g of deuterium and 0.1 g of lithium per ton, every barrel of seawater contains the energy equivalent of almost 30 barrels of oil in deuterium fuel, and about 1/5 barrel of oil in tritium fuel (with tritium produced, or *bred*, from neutron capture on lithium). A volume of seawater equal to the top meter of the oceans would yield enough fuel to power electrical generators for thousands of years at the present consumption rate.

Two methods for producing controlled fusion are popular today, certainly areas of investigation, called magnetic confinement fusion (MCF) and inertial confinement fusion (ICF). Magnetic fusion uses very intense magnetic fields to squeeze a DT plasma to high enough temperatures and densities to ignite and sustain fusion burn. Inertial fusion attempts the same by imploding small pellets, containing DT, with high energy light, ion, or electron beams focused across the pellet. Both are tough problems technologically. With DT fuel, both processes require fuel temperatures in excess of $10^8\ K^o$, and fuel particle densities, n, and confinement times, τ, such that, $n\tau \ \geq \ 10^{15}\ sec/cm^3$. Magnetic fusion operates in a regime, $\tau \approx 1\ sec$, and therefore, $n \approx 10^{15}\ cm^{-3}$. For magnetic confinement fusion, the density is limited by the maximum magnetic field strength that can be generated, often determined by the material strength of the confining vessel. Inertial fusion relies on the mass of the imploding target to provide confinement. For inertial fusion, $\tau \approx 10^{-10}\ sec$, so that, $n \approx 10^{25}\ cm^{-3}$. Again, these are tough technological constraints in the Earth laboratory, but minor operational limitations in the interior furnace of the sun, which keeps bathing the Earth with direct solar energy from fusion processes, some 2 $cal/min\ cm^2$.

Stellar Evolution

The sun is a star with nuclear furnace, like countless others in the Universe. The evolution of nominal stars is detailed by four continuity equations in space and time, much like the equations of hydrodynamics. Star birth occurs following a gravitational instability in interstellar dust clouds, actually a dynamical contraction phase due to gravity against a counteracting pressure gradient. At sufficiently high densities and temperatures in the keV range, thermonuclear reactions occur, with the release of large amounts of energy. In such simplified approach, the star is assumed to be a gaseous sphere, subjected to its own gravity while maintaining spherical symmetry throughout its evolution, from a contracting protostar in interstellar dust, to a very hot, and dense, radiating plasma, to a fuel depleted, dying orb. External forces, magnetic fields, and stellar rotation are not included. The hydrostatic equation, balancing gravity against pressure, takes the form,

$$\frac{\partial P}{\partial m} = -\frac{G_0 m}{4\pi r^4} \tag{66}$$

with P pressure, m the mass inside a stellar sphere at radius, r, and G_0 the gravitational constant. The radial distribution of mass is written,

$$\frac{\partial r}{\partial m} = \frac{1}{4\pi\rho r^2} \tag{67}$$

with ρ the mass density. Conservation of energy requires that the variation in heat content per unit mass be the difference between fusion energy production, ε, and the radiative energy loss, δ,

$$\frac{\partial L}{\partial m} = \varepsilon - \delta \tag{68}$$

with L the stellar luminosity (Hertzsprung-Russell) at distance, r. The temperature gradient, using the radiative Stefan-Boltzmann law, takes the form,

$$\frac{\partial T}{\partial m} = 3\omega L\sigma 16\pi^2 r^4 T^3 \tag{69}$$

with T the temperature, ω the photon opacity, and σ the Stefan-Boltzmann constant. These four equations close upon themselves in the same manner as the hydrodynamic equations of particle, momentum, and energy conservation, and the equation of state. The Hertzprung-Russell luminosity is a photometric measurement of stellar radiative output. The Hertsprung-Russell diagram, as it is called, depicts luminosities as a function of temperature. Variations in Hertzsprung-Russell diagrams for different clusters of stars have aided theories of stellar evolution.

Most stars follow four steps of evolution, namely, gravitational contraction until thermonuclear ignition, expansion due to fusion burn of light elements (hydrogen), lesser expansion due to fusion burn of heavier elements (helium), and final contraction (death) into a white dwarf, neutron star, or black hole (depending on stellar mass) as thermonuclear fuel is depleted, or sometimes enormous explosion (nova). Lightweight stars, with masses less than 4 times the solar mass, usually die as white dwarfs (including the sun). Middleweight stars, up to about 8 solar masses, because they are thought to burn carbon later in their evolution, may die as white dwarfs, or possibly explode as nova and supernova. Heavyweight stars, beyond 8 solar masses, may also explode, but can degenerate (burn) into cold neutron stars or black holes. Neutron stars are very dense objects, essentially compressed neutrons, supported against gravitational collapse by neutron degeneracy (quantum exclusion limit) pressure, while black holes are collapsed gravitational fields, so strong that light emerging from within is completely trapped by gravity, acccording to general relativity. Matter densities in such stellar objects are enormous, on the order of ton/cm^3.

Elementary Particle Interactions

Stellar interactions of enormous proportions are driven mainly by gravity. While such interactions on the cosmological scale are beyond imagination, there exist interactions that are up to 10^{36} stronger than the gravitational forces compressing massive stellar objects, the so called strong, weak, and electromagnetic forces. Elementary particle physics deals with all four at a fundamental level, but a modern focus has been the latter three, namely, strong, weak, and electromagnetic interactions.

The past 40 years have witnessed an explosion of experimental particle data, gathered from high energy accelerators and outer space. Information has been integrated in a consistent picture of elementary particle interactions. Particles are classified in distinct categories. Particles of spin 1/2, with weak and electromagnetic interactions, are called leptons. Leptons include electrons, muons, neutrinos, and their antiparticles. Masses typically range from .511 MeV (electron) possibly up to 1,800 MeV (τ lepton). Neutrinos are massless. Particles with strong interactions, including weak and electromagnetic, are called hadrons. Integer spin hadrons are mesons, while half integer spin hadrons are baryons. Masses of hadrons range from 135 MeV (pion) up to as high as 10,200 Mev (short lived resonances). Baryon numbers are conserved in all interactions.

Meson numbers are not. Hadrons include protons, neutrons, pions, kaons, short lived resonances, and their antiparticles.

The long range forces of gravity and electromagnetism account for large scale macroscopic phenomena, like planetary attraction and charged particle scattering. The short range strong and weak forces account for microscopic phenomena, such as nucleon binding, radioactive nuclear transmutation, and hadron decay into leptons and photons. Strengths of the fundamental forces inversely as their ranges, in order, strong, weak, electromagnetic, and gravitational, and roughly in the ratio, $10^{36} : 10^{22} : 10^{10} : 1$.

One very interesting aspect of elementary particle interactions is matter-antimatter annihilation, and particularly proton-antiproton annihilation. Antiprotons are negatively charged protons, with the same mass and spin. Proton-antiproton annihilation in matter is one of the most energetic reactions observed routinely in high energy physics, some 1.88 GeV per annihilation. Antiprotons, as negatively charged protons, continuously slow down in matter until they are stopped and captured on the surface of a nucleus by a proton, in which case, both proton and antiproton annihilate into gammas, pions, and other shortlived particles. When an antiproton annihilates at rest on the surface of an actinide nucleus (such as uranium and plutonium) many fragments and neutrons are also produced, following direct reaction, nuclear evaporation, and fission processes, along with production of high energy gammas and pions. Collectively, these processes have been termed antiproton fission, for simplicity, because many neutrons are produced as the end result of all reactions. Recent experiments suggest that as many as 15 to 20 neutrons are emitted following antiproton annihilation on U^{238}, that their distribution is peaked near 5 MeV in energy, and that a sizeable fraction (45%-75%) of the annihilation energy (1.88 GeV per annihilation) is deposited locally in the U^{238}. Using hybrid fission-fusion capsules in a pulsed power propulsion engine, it has been estimated theoretically that 8 mg of antiprotons could drive a 10 ton rocket payload to Mars, and back, in 3 months. While technology for producing 8 mg of antiprotons is nonexistent today, the energy densities of antiproton fuels are more than fascinating, and the possibilities more than imagination.

Potential schemes for employing antiproton-proton annihilation as a driver for space propulsion, power generation, condensed matter physics experiments, biomedical treatment, and others, enter the realm of possibility with the advent of portable storage traps (Penning) and related proof-of-principle storage experiments at the European Center For Nuclear Research (CERN) Low Energy Antiproton Ring (LEAR). As many as 10^6 antiprotons have been trapped in Penning traps, and an upper theoretical limit is near 10^{12} antiprotons. Such technology is growing and will port to other interesting systems and experiments, and many new applications are developing.

EXAMPLE PROBLEMS

1. *What does a wrist thermometer of mass,* $m = 10\ g$*, weigh,* w*?*
2. *What does a 1.5 lb abalone iron weigh,* w*?*
3. *What is the density of fresh water,* ρ*, of weight,* $w = 31.2\ lbs$*, occupying* $.5\ ft^3$*?*
4. *What is the density of salt water,* ρ*, of mass,* $m = 2050\ kg$*, occupying* $2.0\ m^3$*?*
5. *A spear gun propels a lock tip shaft at speed,* $v = 34\ ft/sec$*. How long before the shaft impales a target grouper 9* ft *away?*
6. *What is the average speed of a Zodiac covering distance,* $ds = 23\ miles$*, in time,* $dt = 30\ min$*? Docking, the Zodiac stops in time,* $dt = 5.6\ sec$*. What is the magnitude of the deceleration,* a*?*
7. *What is the change in speed,* dv*, of a hydroplane accelerating for time,* $dt = 6\ sec$*, with acceleration,* $a = 24\ ft/sec^2$*?*
8. *A diver surfacing from 150* fsw *covers the first 90* fsw *in 90 sec, and the remaining 60* fsw *in 30 sec. What is the average ascent rate,* r*?*

9. *A submersible of mass, m, moves underwater with speed, v. If the speed is doubled, with is the increase in kinetic energy, ΔK, of the submersible? If the speed is tripled, what is the change in momentum, Δp?*

10. *What are the momentum, p, and kinetic energy, K, of a light diver propulsion vehicle (DPV), $m = 32\ kg$, moving with velocity, $v = 6\ m/sec$? What is the force, F, required to stop it in 8 sec?*

11. *What is the increase in potential energy, U, for a diver of weight, $mg = 150\ lbs$, who ascends from $60\ fsw$ to the surface?*

12. *A diver inflates his BC at depth, $d = 10\ msw$, to approximately $.015\ m^3$. How much work, dW, does the diver do?*

13. *An $80\ kg$ diver giant strides from the deck of a boat into the water $2.7\ m$ below, taking $.74\ sec$ to hit the surface. What is the power, W, generated by the fall? What is the kinetic energy, K, on impact, neglecting air resistance?*

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15. *What is the energy, ϵ, of a photon moving at the speed of light, c, and frequency, $f = 8.2 \times 10^{14}\ sec^{-1}$? What is the corresponding photon wavelength, λ?*

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17. *What is the critical angle, ϕ_c, at the air-water interface, that is, in taking, $n_{air} = 1.0$, and, $n_{water} = 1.33$?*

18. *What is the magnification, μ, and foreshortening, σ, across the quartz-air interface for an object in quartz, viewed in air?*

19. *What is the magnification, μ, and foreshortening, σ, for an object in air, viewed in quartz?*

20. *A coral head appears, $h_{wat} = 8\ ft$, tall, and, $s_{wat} = 6\ ft$, away in Truk Lagoon. What are the actual height, h, and distance, s?*

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28. *A welding thermometer is constructed using changes in resistance to calibrate temperature changes. If the thermometer is logarithmic in response, what is the temperature,* T*, at resistance,* $X = 60$ *ohms, for fixed points,* $T_f = 500\, C^o,\ X_f = 80$ *ohms, and* $T_i = 100\, C^o,\ X_i = 20$ *ohms?*

29. *If an* $i = .8$ *amp current is passed over the* $R = 60$ *ohm resistor in the above welding thermometer, what is the corresponding potential drop,* V*?*

30. *Sunlight striking the shallow azure water off the coast of Cozumel delivers,* $\Gamma = 2\ cal/m^2$*, to the surface. If,* $\rho = .02$*, is reflected, and,* $\tau = .04$*, is transmitted, what fraction,* α*, is absorbed? What is the magnitude,* Γ_r*, of the reflected radiation?*

31. *What is the horizontal force,* F*, necessary to drag a* $16.5\ kg$ *scuba tank across a flat iron plate at a fill station? How much work,* dH*, is done in moving the tank a distance,* $ds = 12\ m$*?*

32. *What is the change in internal energy,* dU*, of air in a compressor heated an amount,* $dQ = 100\ cal$*, while doing piston expansion work,* $dW = 165\ j$*?*

ADDITIONAL READING

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PROBLEM SOLUTIONS

1. What does a wrist thermometer of mass, $m = 10\ g$, weigh, w?

$$w = mg$$

$$w = 10 \times 980\ dynes = 9.8 \times 10^3\ dynes$$

2. What does a 1.5 lb abalone iron weigh, w?

$$w = 1.5\ lb$$

3. What is the density of fresh water, ρ, of weight, $w = 31.2\ lbs$, occupying .5 ft^3?

$$\rho = \frac{w}{V} = \frac{31.2}{.5}\ lb/ft^3 = 62.4\ lb/ft^3$$

4. What is the density of salt water, ρ, of mass, $m = 2050\ kg$, occupying 2.0 m^3?

$$\rho = \frac{m}{V} = \frac{2050.}{2.0}\ kg/m^3 = 1025\ kg/m^3$$

5. A spear gun propels a lock tip shaft at speed, $v = 34\ ft/sec$. How long before the shaft impales a target grouper 9 ft away?

$$v = \frac{ds}{dt}\ \ ,\ \ dt = \frac{ds}{v} = \frac{9}{34}\ sec = .26\ sec$$

6. What is the average speed of a Zodiac covering distance, $ds = 23\ miles$, in time, $dt = 30\ min$?

$$v = \frac{ds}{dt} = \frac{23}{.5}\ mi/hr = 46\ mi/hr$$

Docking, the Zodiac stops in time, $dt = 5.6\ sec$. What is the magnitude of the deceleration, a?

$$a = \frac{dv}{dt} = \frac{-46}{5.6} \times \frac{5280}{3600}\ ft/sec^2 = -12\ ft/sec$$

7. What is the change in speed, dv, of a hydroplane accelerating for time, $dt = 6\ sec$, with acceleration, $a = 24\ ft/sec^2$?

$$dv = adt = 24 \times 6\ ft/sec = 144\ ft/sec$$

8. A diver surfacing from 150 fsw covers the first 90 fsw in 90 sec, and the remaining 60 fsw in 30 sec. What is the average ascent rate, r?

$$r = \frac{ds}{dt} = \frac{150}{90+30}\ fsw/sec = 1.25\ fsw/sec$$

9. A submersible of mass, m, moves underwater with speed, v. If the speed is doubled, with is the increase in kinetic energy, ΔK, of the submersible?

$$\Delta K = \frac{1}{2}m(2v)^2 - \frac{1}{2}mv^2 = \frac{3}{2}mv^2$$

If the speed is tripled, what is the change in momentum, Δp?

$$\Delta p = 3mv - mv = 2mv$$

10. What are the momentum, p, and kinetic energy, K, of a light diver propulsion vehicle (DPV), $m = 32\ kg$, moving with velocity, $v = 6\ m/sec$?

$$p = mv = 32 \times 6\ kg\ m/sec = 192\ kg\ m/sec$$

$$K = \frac{1}{2}mv^2 = \frac{1}{2} \times 32 \times 36\ kg\ m^2/sec^2 = 1.15\ \times 10^3\ j$$

What is the force, F, required to stop it in 8 sec?

$$dp = 192\ kg\ m/sec\ \ ,\ \ dt = 8\ sec$$

$$F = \frac{dp}{dt} = \frac{192}{8}\ kg\ m/sec^2 = 24\ nt$$

11. What is the increase in potential energy, U, for a diver of weight, $mg = 150\ lbs$, who ascends from 60 fsw to the surface?

$$U = mgh = 150 \times 60\ ft\ lb = 9 \times 10^3\ ft\ lb$$

12. A diver inflates his BC at depth, $d = 10\ msw$, to approximately .015 m^3. How much work, dW, does the diver do?

$$dW = PdV$$

$$dW = 20.2 \times 10^4 \times .015\ kg\ m^2/sec^2 = 3.03\ \times 10^3\ j$$

13. An 80 kg diver giant strides from the deck of a boat into the water 2.7 m below, taking .74 sec to hit the surface. What is the power, W, generated by the fall?

$$W = \frac{dH}{dt}\ \ ,\ \ dH = mgh\ \ ,\ \ dt = .74\ sec$$

$$dH = 80 \times 9.8 \times 2.7\ kg\ m^2/sec^2 = 2.13 \times 10^3\ j$$

$$W = \frac{2.13 \times 10^3}{.74}\ j/sec = 2.86 \times 10^3\ watt$$

What is the kinetic energy, K, on impact, neglecting air resistance?

$$v = gt = 9.8 \times .74\ m/sec = 7.3\ m/sec$$

$$K = \frac{1}{2}mv^2 = \frac{1}{2} \times 80 \times 53.3\ j = 2.13 \times 10^3\ j$$

14. A UDT paradiver jumps (no chute) from a USN Seawolf helicopter with initial potential energy, $U_i = 12 \times 10^3\ j$, and zero kinetic energy, $K_i = 0\ j$ (all relative to the surface of the Earth). What is the kinetic energy, K_f, when the paradiver hits the water (neglecting air resistance) in the Gulf of Tonkin?

$$E_i = K_i + U_i = E_f = K_f + U_f$$

$$U_i = 12 \times 10^3\ j\ \ ,\ \ U_f = 0\ j\ \ ,\ \ K_i = 0\ j$$

$$K_f = K_i + U_i - U_f = 0 + 12 \times 10^3 - 0\ j = 12 \times 10^3\ j$$

At some point in the drop, the paradiver gains kinetic energy, $K = 9 \times 10^3\ j$. What is the corresponding potential energy, U?

$$E = K + U = E_i = E_f = 12 \times 10^3\ j$$

$$U = E - K = 12 \times 10^3 - 9 \times 10^3\ j = 3 \times 10^3\ j$$

15. What is the energy, ε, of a photon moving at the speed of light, c, and frequency, $f = 8.2 \times 10^{14}\ sec^{-1}$?

$$h = 6.625 \times 10^{-34}\ j\ sec\ \ ,\ \ f = 8.2 \times 10^{14}\ sec^{-1}$$

$$\varepsilon = hf = 6.625 \times 8.2 \times 10^{-20}\ j = 5.4 \times 10^{-19}\ j = 3.39 \times 10^{-3}\ keV$$

What is the corresponding photon wavelength, λ?

$$c = 2.99 \times 10^{10}\ cm/sec$$

$$\lambda = \frac{c}{f} = \frac{2.99 \times 10^{10}}{8.2 \times 10^{14}}\ cm = 3.6 \times 10^{-5}\ cm$$

16. What is the energy, E, of a lead weight, $m_0 = 1\ kg$, moving at velocity, $v/c = .85$, aboard the Starship Enterprize initiating warp acceleration in the Sea Of Khan?

$$c = 2.99 \times 10^8\ m/sec\ \ ,\ \ m_0 = 1\ kg$$

$$E = \frac{m_0 c^2}{(1 - v^2/c^2)^{1/2}} = \frac{1 \times (2.99 \times 10^8)^2}{(1 - .85^2)^{1/2}} kg\ m^2/sec^2 = 1.72 \times 10^{17}\ j$$

What is the corresponding kinetic energy, K?

$$\gamma = (1 - v^2/c^2)^{-1/2} = .52^{-1/2} = 1.39$$

$$K = (\gamma - 1) m_0 c^2 = .39 \times 1 \times (2.99 \times 10^8)^2\ kg\ m^2/sec^2 = 3.49 \times 10^{16}\ j$$

17. What is the critical angle, ϕ_c, at the air-water interface, that is, in taking, $n_{air} = 1.0$, and, $n_{water} = 1.33$?

$$sin\ \phi_c = \frac{n_{air}}{n_{water}} = \frac{1}{1.33} = .75$$

$$\phi_c = sin^{-1}\ (.75) = 48.5^o$$

18. What is the magnification, μ, and foreshortening, σ, across the quartz-air interface for an object in quartz, viewed in air?

$$\mu = \frac{n_{quartz}}{n_{air}} \ , \ \sigma = \frac{n_{air}}{n_{quartz}}$$

$$\mu = \frac{1.456}{1.000} = 1.456 \ , \ \sigma = \frac{1.000}{1.456} = .687$$

19. What is the magnification, μ, and foreshortening, σ, for an object in air, viewed in quartz?

$$\mu = \frac{n_{air}}{n_{quartz}} \ , \ \sigma = \frac{n_{quartz}}{n_{air}}$$

$$\mu \ = \ .687 \ , \ \sigma = 1.456$$

20. A coral head appears, $h_{wat} = 8 \ ft$, tall, and, $s_{wat} = 6 \ ft$, away in Truk Lagoon. What are the actual height, h, and distance, s?

$$\mu = 1.33 \ , \ \sigma = .75$$

$$h = \frac{h_{wat}}{\mu} = \frac{8}{\mu} \ ft = 6 \ ft$$

$$s = \frac{s_{wat}}{\sigma} = \frac{6}{.75} \ ft = 8 \ ft$$

21. How long, dt, does it take a sound wave to propagate a distance, $d = 10,604 \ m$, in steel?

$$u = 5032 \ m/sec \ , \ dt = \frac{ds}{u} = \frac{10604}{5302} \ sec = 2 \ sec$$

22. A surface tender screams at a diver underwater with acoustical energy, $\varepsilon = 4.1 \ btu$. What is the energy, ε_R, reflected from the surface, and energy, ε_T, transmitted at the surface (absorbed in less than a cm)?

$$\varepsilon_R = R\varepsilon \ , \ \varepsilon_T = T\varepsilon$$

$$\varepsilon_R = .9919 \times 4.1 \ btu = 4.066 \ btu \ , \ \varepsilon_T = .0081 \times 4.1 \ btu = .034 \ btu$$

23. What is the heat flux, ϕ, across a neoprene wetsuit of thickness, $dx = .64 \ cm$, for body temperature of $22.7 \ C^o$ and water temperature of $4.1 \ C^o$?

$$\phi = -K \frac{dT}{dx}$$

$$K = .0004 \ cal/cm \ C^o \ sec \ , \ dx = .64 \ cm \ , \ dT = 22.7 - 4.1 \ C^o = 18.6 \ C^o$$

$$\phi = .0004 \times \frac{18.6}{.64} \ cal/cm^2 \ sec = 1.16 \times 10^{-2} \ cal/cm^2 \ sec$$

24. What heat flux, ϕ, does a *light stick* at 298 K^o emit underwater, and what is the Centigrade temperature, (C^o), of the chemical candle?

$$\phi = \sigma_0 T^4$$

$$T = 298\, K^o \quad , \quad \sigma_0 = 5.67 \times 10^{-8}\, watts/m^2\, K^{o4}$$

$$\phi = 5.67 \times 10^{-8} \times 298^4 watts/m^2 = 447.1\, watts/m^2$$

$$C^o = K^o - 273 = 298 - 273 = 25^o$$

25. If an amount of heat, $dQ = 650\ cal$, raises the temperature of a saline solution, $m = 50\ g$, some, $dT = 14\, C^o$, at constant pressure, what is the specific heat, c_P?

$$c_P = \frac{1}{m}\left[\frac{dQ}{dT}\right]_P = \frac{650}{50 \times 14}\ cal/g\, C^o = .928\ cal/g\, C^o$$

26. How many calories, Q, does it take to just melt 100 g ice?

$$Q = lm \quad , \quad l = 80\ cal/g \quad , \quad m = 100\ g$$

$$Q = 80 \times 100\, cal = 8000\, cal$$

27. What additional amount of heat, dQ, does it take to raise the 100 g of water to its boiling point, $T = 100\, C^o$?

$$dT = 100 - 0 = 100\, C^o \quad , \quad c_P = 1.00\ cal/gC^o$$

$$dQ = mc_P dT = 1.00 \times 100 \times 100\, cal = 10^4\ cal$$

28. A welding thermometer is constructed using changes in resistance to calibrate temperature changes. If the thermometer is logarithmic in response, what is the temperature, T, at resistance, $X = 60\ ohms$, for fixed points, $T_f = 500\, C^o$, $X_f = 80\ ohms$, and $T_i = 100\, C^o$, $X_i = 20\ ohms$?

$$T - T_i = (T_f - T_i)\ \left[\frac{\ln\ X/X_i}{\ln\ X_f/X_i}\right]$$

$$T = 400 \times \left[\frac{\ln\ 60/20}{\ln\ 80/20}\right]\ C^o = 417\, C^o$$

29. If an $i = .8\ amp$ current is passed over the $R = 60\ ohm$ resistor in the above welding thermometer, what is the corresponding potential drop, V?

$$V = iR = .8 \times 60\, volts = 48\, volts$$

30. Sunlight striking the shallow azure water off the coast of Cozumel delivers, $\Gamma = 2\ cal/m^2$, to the surface. If, $\rho = .02$, is reflected, and, $\tau = .04$, is transmitted, what fraction, α, is absorbed?

$$\rho = .02 \quad , \quad \tau = .04$$

$$\rho + \tau + \alpha = 1 \quad , \quad \alpha = 1 - \tau - \rho$$

$$\alpha = 1 - .04 - .02 = .94$$

What is the magnitude, Γ_r, of the reflected radiation?

$$\Gamma_r = \rho\Gamma \quad , \quad \Gamma_r = .02 \times 2\ cal/m^2 = .04\ cal/m^2$$

31. What is the horizontal force, F, necessary to drag a 16.5 kg scuba tank across a flat iron plate at a fill station?

$$\mu = .4 \ \ , \ \ N = mg$$

$$F = \mu N = .4 \times 16.5 \times 9.8 \, kg \ m/sec^2 = 64.7 \ nt$$

How much work, dH, is done in moving the tank a distance, $ds = 12 \ m$?

$$dH = F ds = 64.7 \times 12 \ j = 776.4 \ j$$

32. What is the change in internal energy, dU, of air in a compressor heated an amount, $dQ = 100 \ cal$, while doing piston expansion work, $dW = 165 \ j$?

$$dQ = 100 \times 4.19 \ j = 419 \ j \ \ , \ \ dW = 165 \ j$$

$$dU = dQ - dW = 419 - 165 \ j = 254 \ j$$

BIOSKETCHES

Bruce Wienke is a Program Manager in the Nuclear Weapons Technology/ Simulation And Computing Office at the Los Alamos National Laboratory (LANL), with interests in computational decompression and models, gas transport, and phase mechanics. He authored *Physics, Physiology And Decompression Theory For The Technical And Commercial Diver, High Altitude Diving, Basic Diving Physics And Applications, Diving Above Sea Level, Basic Decompression Theory And Application*, and some 200 technical journal articles. Diving environs include the Caribbean, South Pacific, Asia, inland and coastal United States, Hawaii, and polar Arctic and Antarctic in various technical, scientific, military, and recreational activities. He functions on the LANL Nuclear Emergency Strategy Team (NEST), in exercises often involving Special Warfare Units, above and underwater. He heads Southwest Enterprises, a consulting company for computer research and applications in wide areas of applied science and simulation.

He is an Instructor Trainer with the National Association Of Underwater Instructors (NAUI), has served on the Board Of Directors (Vice Chairman for Technical Diving, Technical and Decompression Review Board Member), is a Master Instructor with the Professional Association Of Diving Instructors (PADI) in various capacities (Instructor Review Committee), is an Institute Director with the YMCA, and is an Instructor Trainer with Scuba Diving International/Technical Diving International (SDI/TDI).

Wienke, a former dive shop owner in Santa Fe, presently works with DAN on applications of high performance computing and communications to diving, and is a Regional Data Coordinator for Project Dive Safety. SCUBAPRO, SUUNTO, ABYSMAL DIVING, and ATOMICS engage (or have) him as Consultant for meter algorithms. He is the developer of the Reduced Gradient Bubble Model (RGBM), a dual phase approach to staging diver ascents over an extended range of diving applications (altitude, nonstop, decompression, multiday, repetitive, multilevel, mixed gas, and saturation). The SUUNTO VYPER dive computer incorporates the RGBM into staging regimens, particularly for recreational diving (including nitrox). ABYSS, a commercial software product, features some of the RGBM dynamical diving algorithms developed by him for Internet users and technical divers. He is also Associate Editor for the International Journal Of Aquatic Research And Education, and is a former Contributing Editor of *Sources*, the NAUI Training Publication.

Wienke received a BS in physics and mathematics from Northern Michigan University, an MS in nuclear physics from Marquette University, and a PhD in particle physics from Nothwestern University. He is a member of the Undersea And Hyperbaric Medical Society (UHMS), American Physical Society (APS), Society Of Industrial And Applied Mathematics (SIAM), and the American Academy Of Underwater Sciences (AAUS).

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Tim O'Leary is Director of NAUI Worldwide Technical Training Operations, Course Director for NAUI Worldwide, Inspector Trainer for PSI, and President of American Diving And Marine Salvage. He has spoken at many underwater symposiums, as well as contributing to recreational and technical periodicals.

He has dived in Asia, South Pacific, North Sea, Mediteranian, Mexico, Central and South America, and the United States as both a mixed gas Commercial Diver and technical diving Instructor Trainer.

O'Leary received a BS in zoology from Texas AM University, a DMT and CHT from Jo Ellen Smith Medical Ceneter at the Baromedical Research Institute. He has worked as a Commercial Diving Instructor at the Ocean Corporation, a Saturation Diver, Gas Rack Operator, Saturation Supervisor, and Chamber Supervisor for many of the world's commercial diving companies. He currently serves as a Consultant for the offshore oil industry, and is a Level III NDT Technician.

O'Leary is a member of the Undersea And Hyperbaric Medical Society (UHMS), Society Of Naval Architects And Marine Engineers (SNAME), National Association Of Diver Medical Technicians (NADMT), and is an Admiral in the Texas Navy.

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